

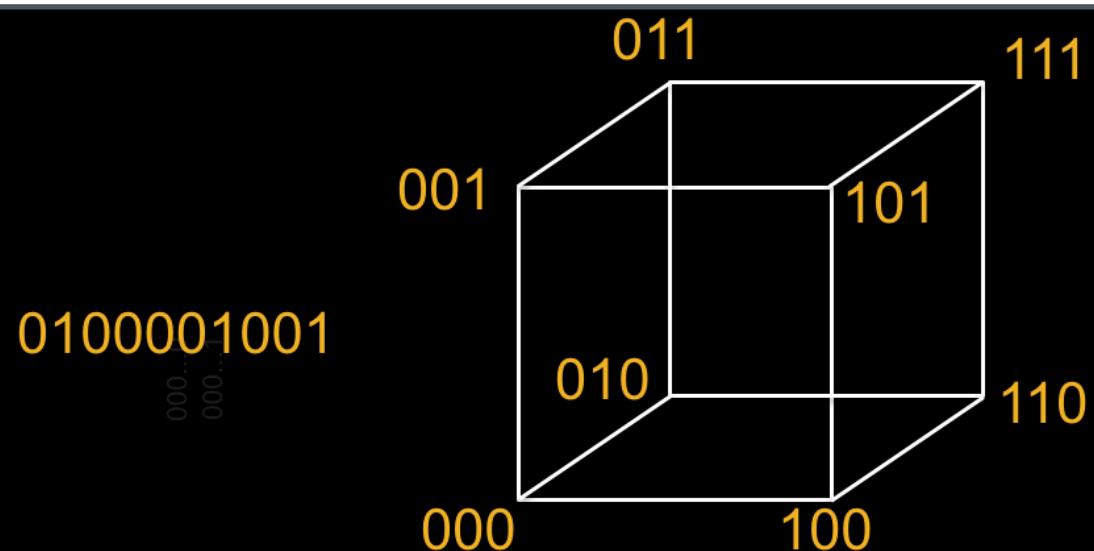
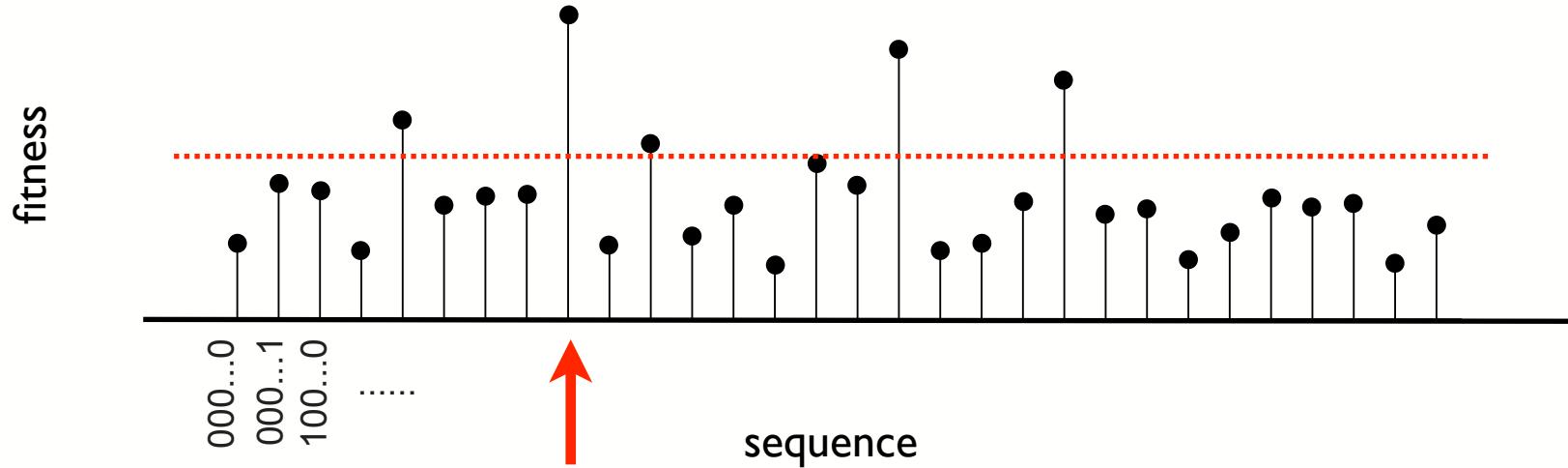
Selection Theory II

Chris Kempes



- What are the basic ways in which we think about evolution?

Fitness Landscape



Quasispecies equation

$$\frac{dx_i}{dt} = \sum_{j=1}^n x_j f_j q_{j,i} - \phi x_i$$

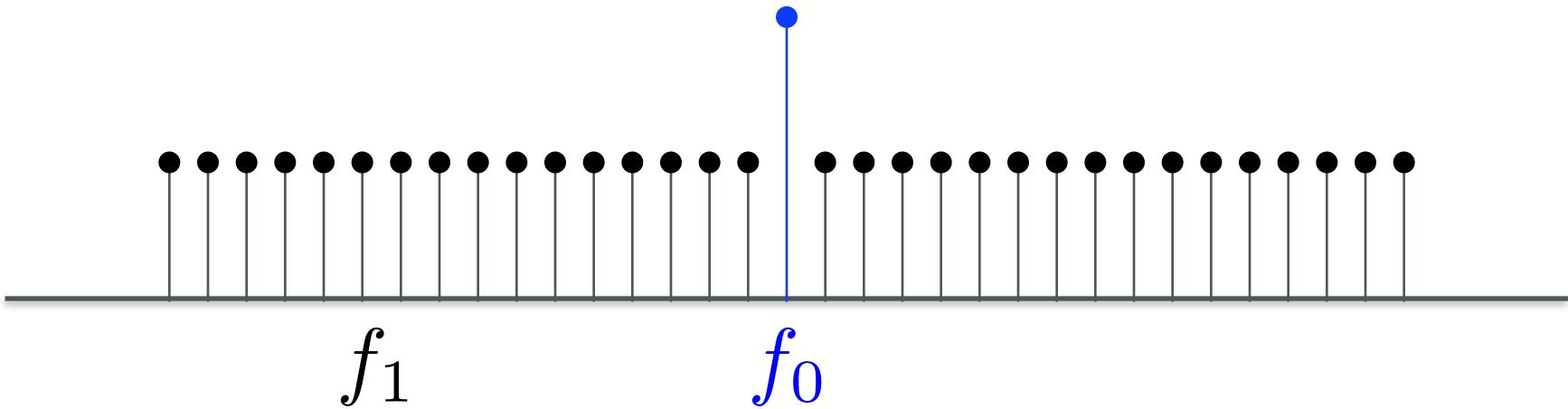
$$\phi = \sum_{i=1}^n x_i f_i$$

x_i Frequency of a given sequence i

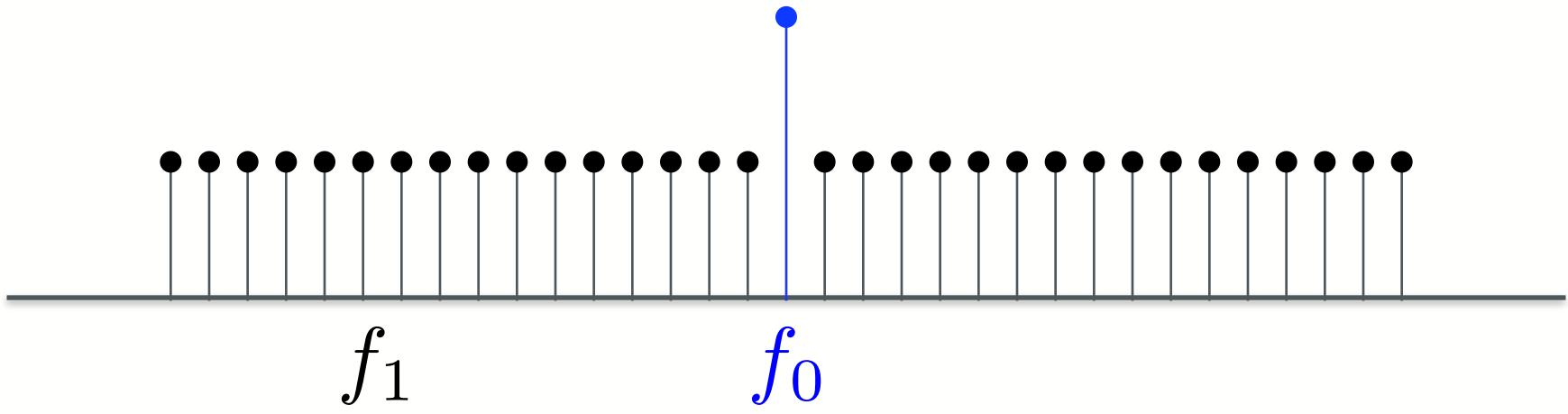
f_i Fitness of a given sequence i
(e.g. relative growth rate)

$q_{i,j}$ Probability of mutating
from sequence i to sequence j

Steady State Solutions



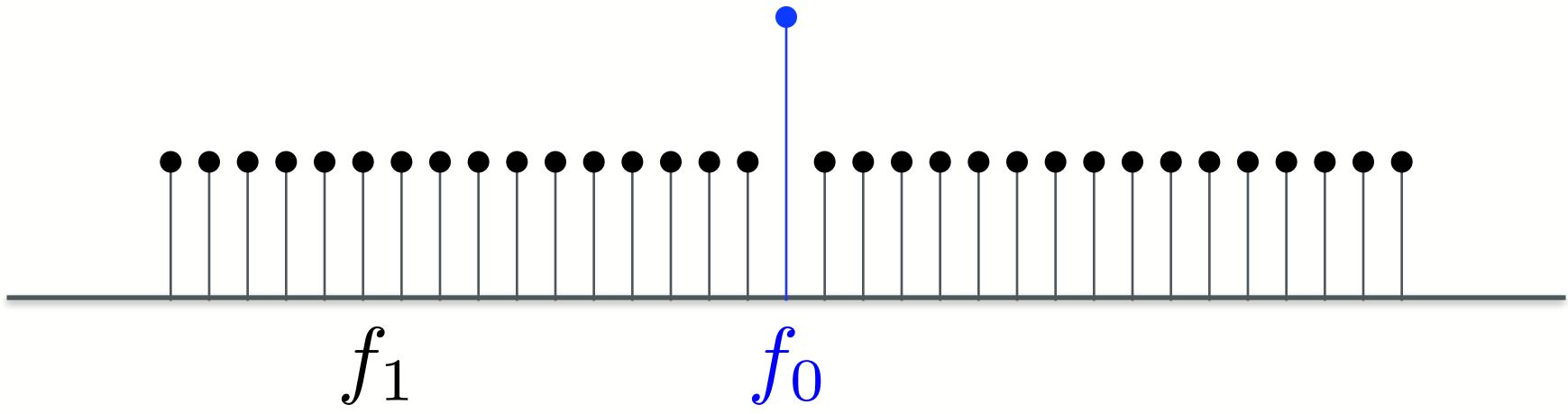
Steady State Solutions



$$f_0 > 1$$

$$f_1 = 1$$

Steady State Solutions

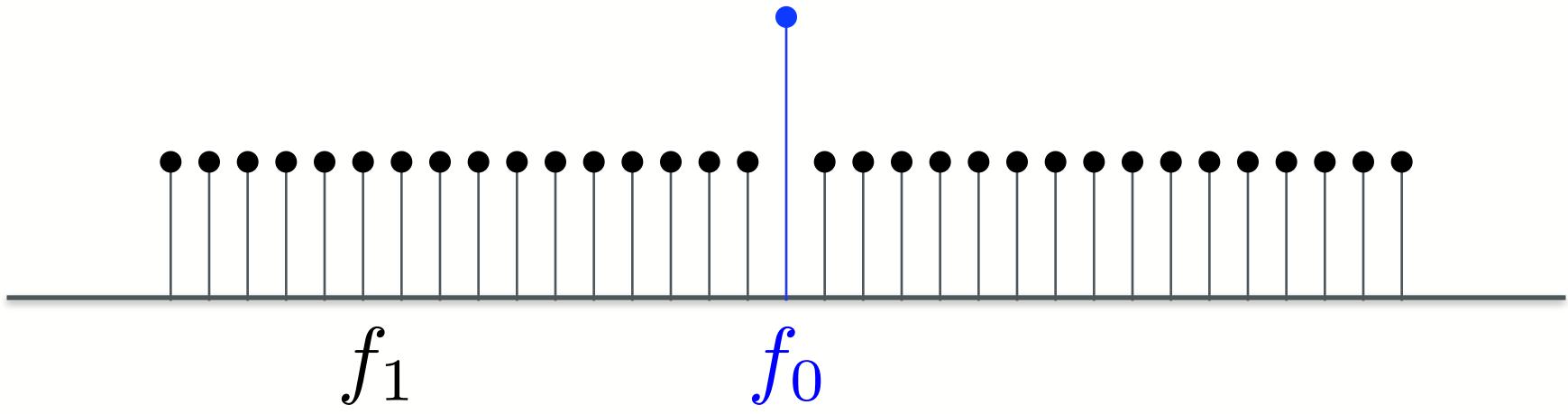


$$f_0 > 1$$

$$q_{0,0} = (1 - u)^L$$

$$f_1 = 1$$

Steady State Solutions



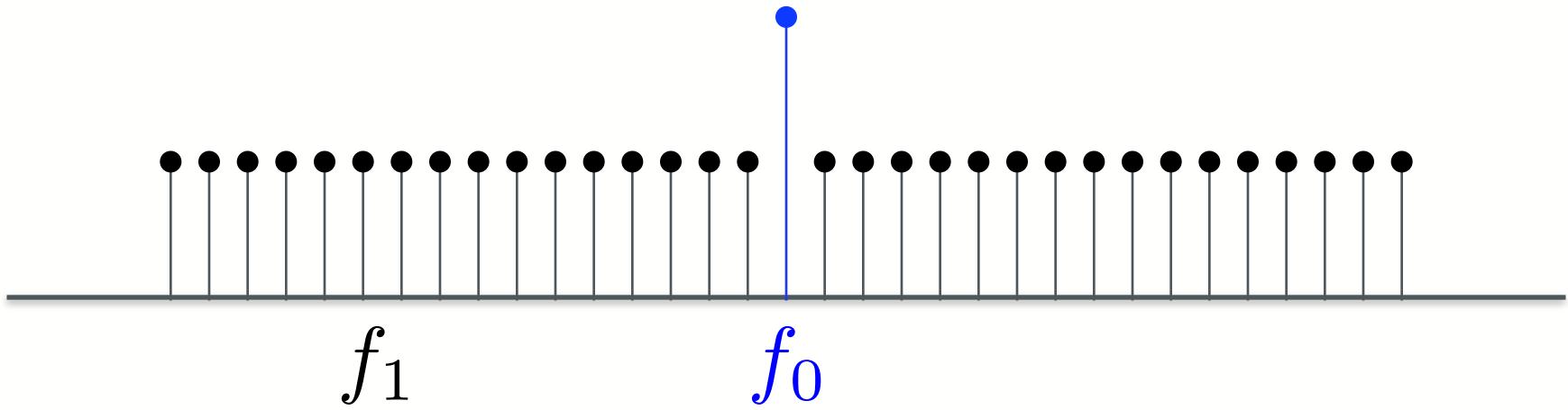
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Steady State Solutions



$$f_0 > 1$$

$$q_{0,0} = (1 - u)^L$$

$$f_1 = 1$$

$$q_{0,1} = 1 - q_{0,0}$$

$$q_{1,0} = 0$$

$$q_{1,1} = 1$$

Steady State Solutions

$$\frac{dx_i}{dt} = \sum_{j=1}^n x_j f_j q_{j,i} - \phi x_i$$

$$\phi = \sum_{i=1}^n x_i f_i$$

Steady State Solutions

$$\frac{dx_i}{dt} = \sum_{j=1}^n x_j f_j q_{j,i} - \phi x_i \quad f_0 > 1$$

$$f_1 = 1$$

$$\phi = \sum_{i=1}^n x_i f_i$$

Steady State Solutions

$$\begin{aligned}\frac{dx_i}{dt} &= \sum_{j=1}^n x_j f_j q_{j,i} - \phi x_i & f_0 > 1 & \quad q_{0,0} = (1-u)^L \equiv q \\ \phi &= \sum_{i=1}^n x_i f_i & f_1 = 1 & \quad q_{0,1} = 1 - q \\ &&& \quad q_{1,1} = 1 \\ &&& \quad q_{1,0} = 0\end{aligned}$$

Steady State Solutions

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$$\frac{dx_0}{dt} = x_0 f_0 q - x_0 \phi$$

Steady State Solutions

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$$\frac{dx_0}{dt} = x_0 f_0 q - x_0 \phi$$

$$\frac{dx_1}{dt} = x_0 f_0 (1-q) + x_1 - x_1 \phi$$

Steady State Solutions

$$\begin{aligned}\frac{dx_i}{dt} &= \sum_{j=1}^n x_j f_j q_{j,i} - \phi x_i & f_0 > 1 & \quad q_{0,0} = (1-u)^L \equiv q \\ \phi &= \sum_{i=1}^n x_i f_i & f_1 = 1 & \quad q_{0,1} = 1 - q \\ &&& \quad q_{1,1} = 1 \\ &&& \quad q_{1,0} = 0\end{aligned}$$

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Steady State Solutions

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$$x_0 + x_1 = 1$$

Steady State Solutions

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$$\phi = f_0 x_0 + x_1$$

$$x_0 + x_1 = 1$$

$$\frac{dx_0}{dt} = x_0 [f_0 q - 1 - x_0 (f_0 - 1)]$$

Steady State Solutions

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$$x_0^* = \frac{f_0 q - 1}{f_0 - 1}$$

Steady State Solutions

$$\frac{dx_0}{dt} = x_0 [f_0 q - 1 - x_0 (f_0 - 1)]$$

$$x_0^* = \frac{f_0 q - 1}{f_0 - 1}$$

$$x_0^* > 0 \quad f_0 q > 1$$

Error Threshold

$$f_0 q > 1 \quad \log(f_0) > -L \log(1-u)$$

$$u < \frac{\log(f_0)}{L}$$

